ANALYSIS THE DYNAMIC STABILITY OF PLATE

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Abstract

In this paper a numerical approach to analyze the dynamic stability of a ribbed plate subjected to harmonic loading is presented. The harmonic balance method and the modified Newmark method of direct integration to solve the equations of motion are used. Unstable regions which include resonant frequencies of the external load are discussed. Then the stability loss occurs by flutter, i.e. exponentially growing oscillations.
The modified Newmark direct integration method to analyze the equations of motion is used. Some results which describe the time history of resonance vibrations (unstable) and those nonresonances (stable) are presented. The effects of constraint parameters such a modulation depth β and constraint frequency θ are investigated. Also the initial load and internal damping are taken into account. Physical rigidity of plate may be changed by adding initial tension load, and currently resonance vibration will transform to non-resonance. Stability of the system by means of initial load is the more effective, the larger is the system rigidity in the direction of putting the load, as then may be introduced larger value of initial load. Stability of the system by means of initial load is the more effective, the larger is the system rigidity.

1. Introduction

Stability is a resistance of material systems to disturbance, it means conservation of motion (or state of equilibrium) if disturbances are small range. If considered motion (or state of equilibrium) gets significant changes even under small range of disturbance, then we call it unstable motion (or unstable state of equilibrium). In the other ways, when a disturbances caused motion, which increase a distance of mechanical systems from equilibrium state or caused vibration near equilibrium state which increasing its amplitude, then equilibrium state is unstable, aliasing with loss of dynamic stability phenomena.
This phenomena, occurs with different values of loading forces, unnecessary are greater then critical loading, more particularly under harmonic loading. For those cases of loading, analyze the dynamic stability of structure, may based on determining the limits of parametric resonance regions as function of loading values and frequency, so-called Ince-Strutt chart of instability boundaries.

2. Formulation equations of motion using finite element method

Using finite element method we obtain following components equations of motion
- mass matrix $\boldsymbol{M}$

$$\boldsymbol{M}\ddot{\boldsymbol{q}} = \int_{V} \boldsymbol{N}^{T} \rho \boldsymbol{N} dV \ddot{\boldsymbol{q}}\,, \tag{1}$$

- linear stiffness matrix $\boldsymbol{K}$

$$\boldsymbol{K}\boldsymbol{q} = \int_{v} \boldsymbol{B}^{T} D \boldsymbol{B} dV \boldsymbol{q}\,, \tag{2}$$

- geometric stiffness matrix $\boldsymbol{K}_g$

$$\boldsymbol{K}_g \boldsymbol{q} = \int_v \sigma \ \boldsymbol{B}^T \boldsymbol{B} dV \boldsymbol{q} . \tag{3}$$

Rayleigh type viscous damping are used, because for vibrations forced by harmonic signal with θ frequency, near resonance, in which damping is significant we can assume $\theta \approx \omega$ (ω – periodicity of structure), then damping matrix will be as following

$$\boldsymbol{C}\dot{\boldsymbol{q}} = 2\frac{\xi}{\theta}(\boldsymbol{K} - \boldsymbol{K}_g)\dot{\boldsymbol{q}} \ , \tag{4}$$

where: ξ – means true damping to critical damping ratio, also called loss factor.
The equation of motion be as following

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + (\boldsymbol{K} - \boldsymbol{K}_g)\boldsymbol{q} = 0 \tag{5}$$

and describe free vibrations of structure under pre-load loading (stable load) which took into account in geometric stiffness matrix $\boldsymbol{K}_g$ (pre-loading load matrix).

3. Analysis of dynamic stability

In the case when the system load vector is in the form of harmonic function

$$\boldsymbol{P}(t) = \alpha \boldsymbol{P}^s + \beta \boldsymbol{P}^z \cos\theta t \ = \ \alpha \boldsymbol{\Lambda} + \beta \boldsymbol{\Pi} \cos\theta t \ , \tag{6}$$

where: $\alpha \boldsymbol{P}^s$ - constant part of load, $\beta \boldsymbol{P}^z$ - amplitude of variable part of load. Matrixes $\boldsymbol{\Lambda}$ and $\boldsymbol{\Pi}$ are matrixes of geometric rigidity $\boldsymbol{K}_g$, used for introducing external load to the system. Then the equations of motion take the form:

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + (\boldsymbol{K} - \alpha\boldsymbol{\Lambda} - \beta\boldsymbol{\Pi}\cos\theta t)\boldsymbol{q} = 0 \tag{7}$$

and they describe free vibration which is called *parametric free vibration*. For analysis of these equations the method of direct integration and the method of harmonic balance have been used.

3.1. The method of harmonic balance

System of equations (7) has periodic solutions with periods *T* and *2T*, for getting them the odd Fourier series (8) was used (8) (with $4\pi/\theta$ period)

$$\boldsymbol{q} = \sum_{k=1,3,5,...}^{\infty} (\boldsymbol{a}_k \sin\tfrac{1}{2}k\theta t + \boldsymbol{b}_k \cos\tfrac{1}{2}k\theta t) \ , \tag{8}$$

where: $\boldsymbol{a}_k$ and $\boldsymbol{b}_k$ - time independent vectors.
The condition of non-zero solutions is zeroing of determinant (11).
It is similarly when we assume the solution in the form of even Fourier series

$$\boldsymbol{q} = \tfrac{1}{2}\boldsymbol{b}_0 + \sum_{k=2,4,6,...}^{\infty} (\boldsymbol{a}_k \sin\tfrac{1}{2}k\theta t + \boldsymbol{b}_k \cos\tfrac{1}{2}k\theta t) . \tag{9}$$

In this case we assign parametric vibration frequencies on the solution boundary of the system of equations (7) with period T, since series (9) is function with $2\pi/\theta$ period. We get infinite determinant with form (12).

From the above considerations it results that dynamic stability analysis of the system consists in determining limits of resonance regions, i.e. determination of forcing frequency function $\theta = f(\alpha, \beta, \gamma)$ in domain of multipliers of load and damping.

By passing over the damping, the beginning of resonance regions starts with relationship.

$$\theta_{\alpha j} \cong \frac{2\omega_{\alpha j}}{p} , \qquad \begin{pmatrix} j = 1,2,3,\ldots \\ p = 1,2,3,\ldots \end{pmatrix} . \tag{10}$$

For natural frequency $\omega_{\alpha j}$ there is respectively: for $p = 1$ it is $\theta_{\alpha j} = 2\omega_{\alpha j}$ and it is the main resonance (it starts from double natural frequency), for $p = 2$ it is $\theta_{\alpha j} = \omega_{\alpha j}$ - secondary resonance, for $p = 3$ it is $\theta_{\alpha j} = (2/3)\omega_{\alpha j}$ and it is secondary resonance as well, etc.

$$\begin{vmatrix} K-\alpha\Lambda-\frac{9}{4}\Theta^2 M & -\frac{1}{2}\beta\Pi & 0 & -\frac{3}{2}\Theta C \\ -\frac{1}{2}\beta\Pi & K-\alpha\Lambda+\frac{1}{2}\beta\Pi-\frac{1}{4}\Theta^2 M & -\frac{1}{2}\Theta C & 0 \\ 0 & \frac{1}{2}\Theta C & K-\alpha\Lambda-\frac{1}{2}\beta\Pi-\frac{1}{4}\Theta^2 M & -\frac{1}{2}\beta\Pi \\ \frac{3}{2}\Theta C & 0 & -\frac{1}{2}\beta\Pi & K-\alpha\Lambda-\frac{9}{4}\Theta^2 M \end{vmatrix} = 0 , \tag{11}$$

$$\begin{vmatrix} K-\alpha\Lambda-4\Theta^2 M & -\frac{1}{2}\beta\Pi & 0 & 0 & -2\Theta C \\ -\frac{1}{2}\beta\Pi & K-\alpha\Lambda-\Theta^2 M & 0 & -\Theta C & 0 \\ 0 & 0 & K-\alpha\Lambda & -\beta\Pi & 0 \\ 0 & \Theta C & -\frac{1}{2}\beta\Pi & K-\alpha\Lambda-\Theta^2 M & -\frac{1}{2}\beta\Pi \\ 2\Theta C & 0 & 0 & -\frac{1}{2}\beta\Pi & K-\alpha\Lambda-4\Theta^2 M \end{vmatrix} = 0 . \tag{12}$$

4. Determination of resonance frequency regions for plate

It was considered a plate (Fig. 1) supported in a jointed way all over the circumference. Calculation data are as follows: $E = 2.1 * 10^5$ [MPa], $\nu = 0.3$, thickness $\delta = 1$[cm], density $\rho = 7830$ [kg/m^3].

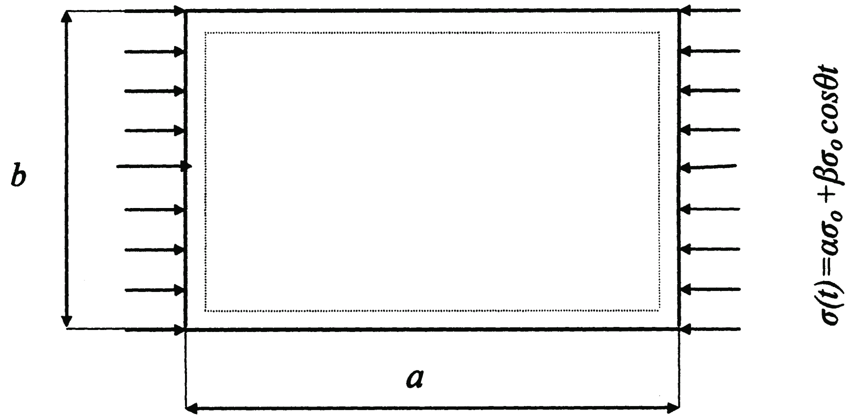


Fig. 1. Plate supported in a jointed way on the circumference

The plate is loaded in its middle surface, the comparative stresses for the plate have been determined from the relationship

$$\sigma_0 = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{\delta}{b}\right)^2, \tag{13}$$

we assume that the plate is under load variable in time, resulting in

$$\sigma(t) = \alpha\sigma_0 + \beta\sigma_0 \cos\theta t\,. \tag{14}$$

For the plate with dimensions *b=1.2*[m], *a=1.8*[m], there have been determined the first four unstable regions, i.e. for *p=1, 2, 3, 4* for the first natural frequency $\omega_{\alpha 1}$, solving determinants (11) and (12). The results are shown on Fig. 2. It shows that the main unstable region *p=1* is definitely larger than secondary regions, taking into account damping with value $\xi = 0.02$ reduces it not much.
Increasing the damping up to value $\xi = 0.2$ results in its reduction together with displacement in direction of larger values of modulation β.

Whereas for the other secondary unstable regions, the influence of damping with value $\xi = 0.02$ is very strong, especially in moving away these regions from frequency axis.

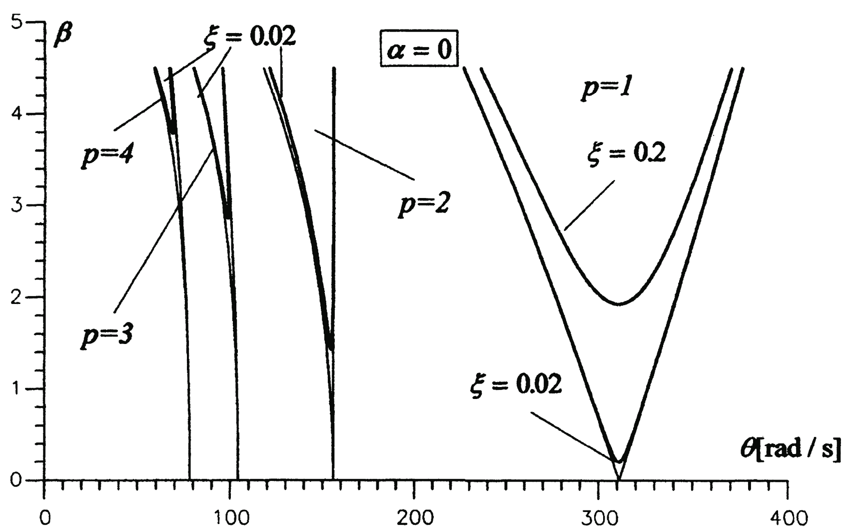


Fig. 2. Limits of regions for p=1, 2, 3, 4 of the first resonance frequency

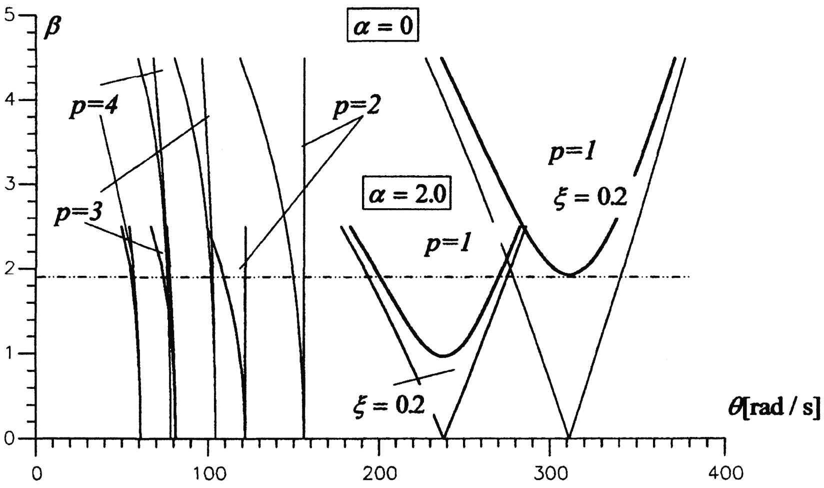


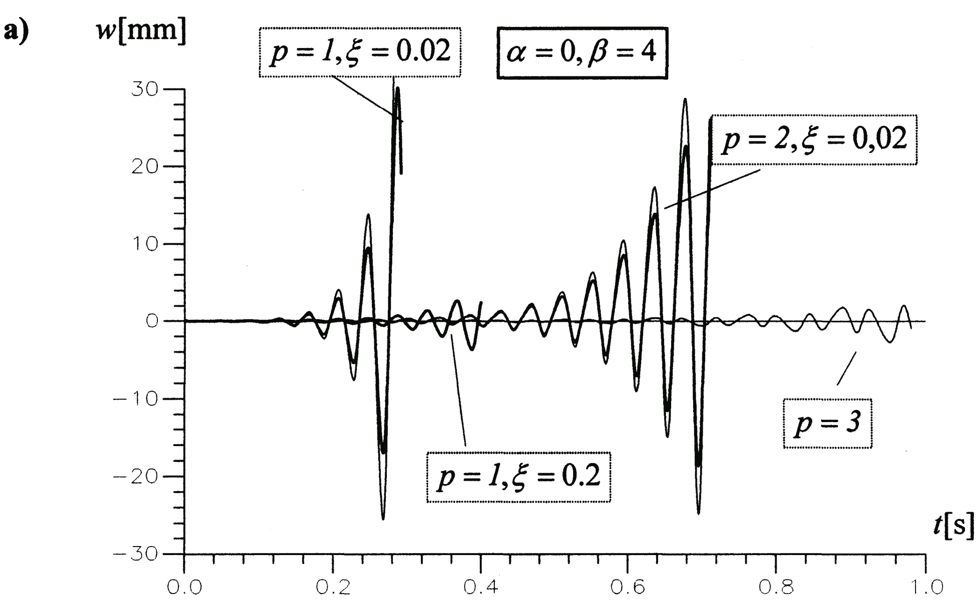
Fig. 3. Limits of regions for p=1, 2, 3, 4 of the first resonance frequency at initial load $\alpha = 2.0$

On Fig.3 on the background of regions without initial load $\alpha = 0$ are put results with $\alpha = 2.0$.

5. Determination of vibration diagrams

Equations of motion (7) have been solved with Newmark method of direct integration adapted to variable rigidity matrix. There have been determined vibration diagrams (Fig. 4) of the plate middle point, taking forcing frequencies θ from unstable regions of resonance frequencies (Fig. 2), for modulation value $\beta = 4$.
Analyzing the diagrams of plate vibration it can be seen that in relatively short time there take place resonance vibration amplitudes, for resonances *p* = *1* and *2* (Fig. 4a). Also, after a little longer time run there take place resonance vibration for resonances *p* = *3* (Fig. 4b).

Then there were analyzed plate vibration courses at presence of initial load α, getting resonance vibration (*p* = *1*) shown on Fig. 5. Respective vibration courses are determined at the following load parameters (for each was taken $\beta = 2$): *diagram* **a** for $\alpha_x = \alpha_y = 0$; *diagram* **b** for $\alpha_x = 2$; *diagram* **c** for $\alpha_y = 1$; *diagram* **d** for $\alpha_y = -1$; *diagram* **e** for $\alpha_y = -2$; - taking forcing frequencies θ, respectively from the resonance regions of the above mentioned load cases.



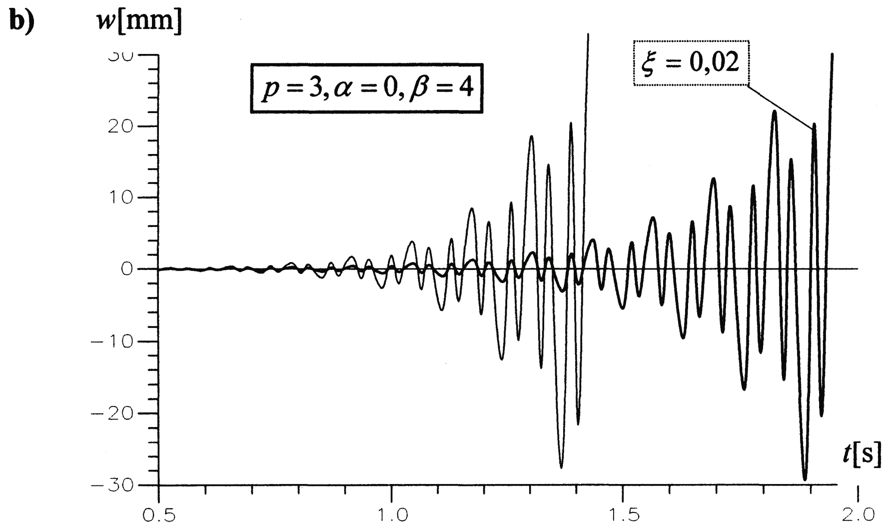


Fig. 4. Plate vibration diagrams

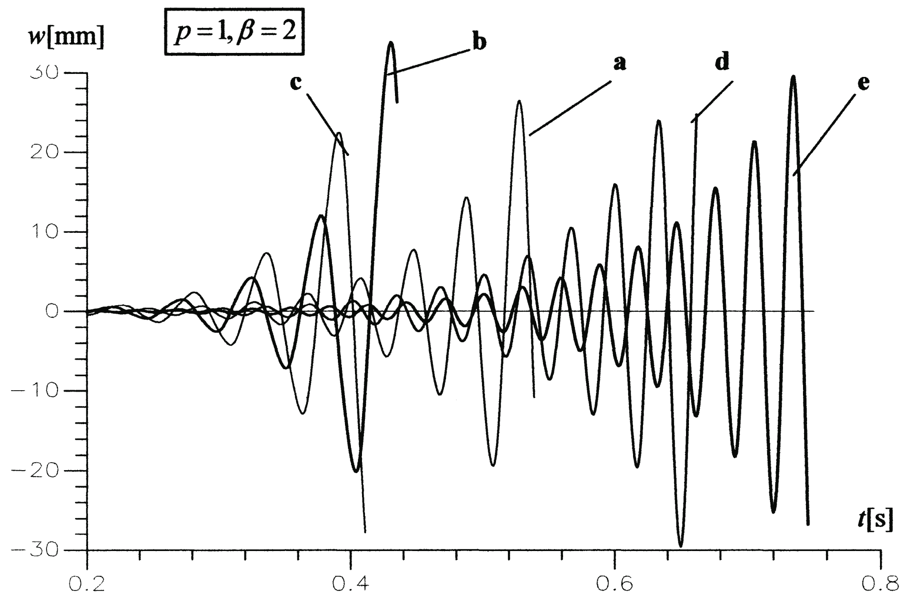


Fig. 5. Plate vibration diagrams at different initial load

Physical rigidity of plate may be changed by adding initial tension load, and currently resonance vibration will transform to non-resonance. Stability of the system by means of initial load is the more effective, the larger is the system rigidity in the direction of putting the load, as then may be introduced larger value of initial load.

References

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